Low-noise preamplifier with input and feedback transformers for low source resistance sensors

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We describe the design, schematics, and performances of a very-low-noise, low-frequency preamplifier. It operates in the 5 Hz–100 kHz range and offers an input equivalent voltage noise density as low as 65 pV/√Hz; the current noise increases with frequency and settles to about 1.5 pA/√Hz at 100 kHz. The amplifier uses input and feedback signal transformers and operates in full differential mode; a -16 dB common-mode rejection ratio is achieved at line frequency. Design methodology is applicable to other ranges of frequencies.

I. INTRODUCTION

Some low-level, low-impedance sensors, typically dCSQUIDs, have to be connected to low-noise, low-resistance preamplifiers in order to take profit of their high intrinsic resolution. Equivalent input noise voltage of a few 10 pV/√Hz and optimal noise resistance of a few ohms are typical values which have to be achieved. In order to reach such a level of performance, two techniques are currently used. In the first one, the sensor is associated to a coil to form a high-quality factor LC tank circuit; it is therefore frequency selective. On the contrary, the other one offers a much wider frequency range since it makes use of an up-signal transformer.

We have designed an input and feedback transformer preamplifier whose input noise voltage is as low as 65 pV/√Hz and whose frequency range extends at a low source impedance from 5 Hz to about 100 kHz. In Sec. II we discuss within some calculation of the amplifier optimization in terms of noise, bandwidth, and long-term gain stability. The second part gives a closer analysis of the designed structure while the last part presents experimental results on both the signal and noise points of view.

II. THE INPUT TRANSFORMER COUPLED FEEDBACK PREAMPLIFIER: PRINCIPLE AND PERFORMANCES

Preamplifier noise performances come mainly from their first stage contribution; by an appropriate choice of active devices and bias conditions, optimal source resistance of a few 10 Ω can be obtained. However, for sensors of much lower source resistances, input transformer coupling must be introduced if a wide frequency response is to be obtained. Characteristics of such an arrangement can be greatly enhanced in terms of gain stability and low cutoff frequency by the introduction of parallel-serial feedback, as shown schematically in Fig. 1(a). Figure 1(b) gives a more detailed description of this circuit and allows calculations of both the input–output transfer function and noise characteristics.

Let us first express the amplifier output voltage \(v_0\) as a function of the three independent generators \(e_S\), \(e_F\), and \(i_A\); it becomes

\[
u_0 = \frac{jAMF_0/Z_1}{1 + jH_0(AFMF_0/Z_1)} \times \left(\frac{Z_1}{jM_{F0}} - \frac{M_F^2 + Z_2}{jM_{F0}} i_A\right),
\]

where

\[
Z_1 = R_S + R_F + R_0 + jL_F \omega,
\]

\[
Z_2 = R_F + jL_F \omega,
\]

and

\[
M_F = \sqrt{L_{F1} L_{F2}} = N_F L_{F1}.
\]

In expression (1), \(R_F\) (respectively \(R_{F1}\)) and \(L_F\) (respectively \(L_{F1}\)) are the resistance and self-inductance of the transformer windings with a coupling factor assumed to be unity and a transformation ratio noted as \(N_F\); \(R_S\) stands for the source resistance and \(A\) is the amplifier gain.

A. Signal analysis

From expression (1) and for \(e_A = i_A = 0\), it is clear that the amplifier transfer function is of a first-order high-pass type. As expected, at high frequency and high loop gain, the \(v_0/e_S\) ratio is only dependent on \(H_0\) while the low cutoff frequency, which is expressed as

\[
f_0 = \frac{R_S + R_F}{2 \pi L_{F1} (1 + AH_0 N_F)},
\]

is, compared to its open-loop value, lowered by the loop gain factor.

B. Noise analysis

From expression (1), the preamplifier equivalent input noise density noted \(e_{in}\) can also be obtained in an easy way. It becomes

\[
e_{in} = \frac{e_S^2}{\alpha}.
\]
FIG. 1. (a) Sketch of the input transformer coupled feedback preamplifier. (b) Detailed view of the preamplifier allowing calculations of the signal and noise characteristics; the generators $e_1$ and $i_4$ are fictitious sources.

\[ i_n^2 = \frac{(e_{\eta_\alpha}^2 + e_{\eta_\beta}^2)}{N_F^2 L_F \omega^2} + \left[ 1 + \left( \frac{R_{F_1}}{N_F^2 L_F \omega} \right)^2 \right] N_F^2 \eta_{n,4} \]  

(5)

Since (4) and (5) are both functions of $e_{\eta_\alpha}$, $e_{\eta_\beta}$, and $i_{n,4}$, some degree of correlation exists between them, but this only matters close to the optimal source impedance and can thus be discarded for simplicity in the further discussion.

In the high-frequency limit, (4) and (5) become

\[ e_{\eta_\alpha}^2 = e_{\eta_\alpha}^2 + e_{\eta_0}^2 + \frac{e_{\eta_\beta}^2}{N_F^2} \]

\[ i_n^2 = 2N_F^2 \eta_{n,4} \]  

(6)

(7)

Compared to the ideal situation which leads as expected to an unmodified amplifier noise energy ($e_{\eta_\alpha}^2$ and an optimal source resistance divided by $N_F$), some additive terms which come in (6) from the input transformer losses and from the feedback equivalent resistance $R_0$ have to be minimized. Such an objective requires a careful design of the input transformer. The magnetic losses can be made negligible by a proper choice of the core pot and ohmic losses minimized, for a given wiring section $S$, resistivity $\rho$, and mean turn length $l$ when both winding sections are equal. In these conditions, the equivalent ohmic resistance of the transformer referred to the input expresses as

\[ R_{TF} = 4K(\rho S)N_F^2 \]

(8)

where $K$ stands for the winding filling factor and $N_F$ for the number of turns of the primary coil.

Moreover, Eq. (2) shows (all others conditions taken constant) that the input self-impedance of the transformer, which varies as $N_F^2$, and as the specific inductance of the core pot, must be chosen as high as possible in order to lower the low cutoff frequency $f_0$. Such a condition goes conversely with the $R_{TF}$ minimization and leads to a compromise. We have chosen to set the transformer input noise contribution,

\[ e_{\eta_\alpha}^2 + (e_{\eta_\beta}^2/N_F^2) \]

at the same level as that of the amplifier one ($e_{\eta_\alpha}^2/N_F^2$), the other terms in (6) being in practice of negligible effect.

From (2) and (8) we obtain, therefore, within these conditions,

\[ N_F^2 \approx 8k_BTl(1 + A\eta_0 N_F) \]

(9)

where $A_{\eta}$ is the specific inductance of the core pot. Equation (9) relates the optimal $N_F$ value to the desired amplifier parameters, operating conditions, and technological constraints. A quick numerical evaluation of (9) shows...
that $N_F$ ranges from 20 to 60 and shows that $e_n = 60 \mathrm{pV}/\sqrt{\text{Hz}}$ can be achieved.

III. THE LOW-NOISE INPUT AND FEEDBACK TRANSFORMER COUPLED DIFFERENTIAL STRUCTURE

Consider now the preamplifier structure shown in Fig. 2(a). It differs from that of Fig. 1 by the addition of a transformer $T_R$ in the feedback loop, which first allows us to operate in a differential mode (a major feature for the detection of low-level signals) and, since $N_F \ll 1$, to easily put the $R_0$ resistance of the feedback path (Fig. 1) in the mΩ range. This feedback transformer $T_R$ also behaves as a high-pass device, which brings some modifications of the low-frequency transfer and noise characteristics, which we briefly review now.

A. Signal analysis

From Fig. 2(b), the input–output transfer function can be obtained easily and expressed as

$$V_o = G_0 p^2 \frac{1 + \frac{1}{(1 + H_0 AN_F N R)}}{p^2 + \alpha \omega_0 p + \omega_0^2},$$

where $p$ is the Laplace complex variable and with

$$G_0 = \frac{AN_F}{1 + H_0 AN_F N R}, \quad H' = \frac{R_B}{R_A + R_B},$$

$$\omega_0^2 = \frac{(R_S + RF_1 + RF_2)(r + R_1)}{L_{R1} L_{F1}(1 + H_0 AN_F N R)}, \quad r = \frac{R_1 R_B}{R_A + R_B},$$

and

$$\alpha = \frac{1}{(R_S + RF_1 + RF_2)(r + R_1)} \times \frac{L_{R1}(R_S + RF_1 + RF_2) + (L_{F1} + L_{R1})(r + R_1)}{L_{F1} L_{R1}(1 + H_0 AN_F N R)}.$$  

Equation (10) shows that the preamplifier transfer function is the sum of a bandpass and of a high-pass second-order-type parts. In the practical design, the bandpass term is of low effect since $L_{R1} \omega / (r + R_1) \gg 2$ in the frequency range of interest; furthermore, a peaked response is expected for a low source resistance up to the critical value defined by the $\alpha - \sqrt{2}$ condition in (13).

B. Noise analysis

In a similar way as in Sec. II B, the effect of the circuit noise sources can be referred back to the preamplifier input. Since the system transfer function is of second-order type, complete expressions are rather complex; therefore, only their asymptotic forms are given in Table I, leaving out a limited frequency range lying in close proximity to $\omega_0$.

Thus, from Table I, the high-frequency input equivalent noise characteristics are expressed as

<table>
<thead>
<tr>
<th>$\omega \rightarrow 0$</th>
<th>$\omega \rightarrow \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{r1}$</td>
<td>$(RF_1 + RF_2) \alpha_i$</td>
</tr>
<tr>
<td>$i_{r1}$</td>
<td>$\alpha_i / N_F$</td>
</tr>
<tr>
<td>$e_{x1}$</td>
<td>$e_{x1}$</td>
</tr>
<tr>
<td>$i_{x1}$</td>
<td>$i_{x1}$</td>
</tr>
<tr>
<td>$e_{r2}$</td>
<td>$e_{r2}$</td>
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<td>$i_{r2}$</td>
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<td>$e_{x2}$</td>
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<tr>
<td>$i_{x2}$</td>
<td>$i_{x2}$</td>
</tr>
</tbody>
</table>

Table I. Input equivalent voltage and current noise densities of the various generators involved in the circuit. For simplicity, only the asymptotic expressions in the low- and high-frequency limits are given.
\[ e_n^2 \omega \to \infty = e_n^2 + \frac{e_n^2}{N_F^2} + e_n^2 + N_F^2 e_n^2 + N_F e_n^2 + e_n^2 \]
\[ + [R_{F1} + R_{R2} + N_F^2 (R_{R1} + r)]^2 N_F^2 e_n^2 \]

and
\[ i_n^2 \omega \to \infty = N_F^2 i_n^2 \]

while in the low-frequency limit it becomes
\[ e_n^2 \omega \to 0 = e_n^2 + e_n^2 + \frac{(R_{F1} + R_{R2})^2}{L_{F1}} \left( \frac{e_n^2 + e_n^2}{N_F^2} \right) \]
\[ + \left( \frac{R_{F1} (R_{F1} + R_{R1})}{N_F^2} \right)^2 N_F^2 e_n^2 \]

and
\[ i_n^2 \omega \to 0 = \frac{e_n^2 + R_{F2} e_n^2}{(L_{F1} \omega N_F)^2} \cdot \]

Equation (14) shows that in order to obtain the best results in the high-frequency limit, the feedback transformer has to offer an input equivalent resistance as weak as possible, its characteristics must then lie in the same range as that of the input transformer, which leads to a \( N_F N_F \) product close to unity. Furthermore, since the resistive feedback path brings a \( N_F^2 e_n^2 \) term, its equivalent resistance must also be taken low.

Finally, Eq. (16) indicates that several terms increase from their high-frequency limit as \( [(R_{F1} + R_{R1})/L_{F1}]^2 \); from Eq. (2) it is clear that this crossover point corresponds to the \( R_S = 0 \) open-loop low cutoff frequency of the preamplifier which has to be minimized.

IV. PREAMPLIFIER DESCRIPTION AND CHARACTERISTICS

We shall now turn to the full description of the preamplifier schematics shown in Fig. 3. Besides the requirements resulting from the above analysis, the design has also taken into account the high-frequency limitations coming mainly from the input transformer self-resonance.

Let us first consider the technological specifications of the input and feedback transformers. They are based on the Siemens 41 \( \times \) 25 pot cores made of low-loss N 48 ferrite material. The total air gap of the pot cores was initially 0.18 mm and the corresponding specific inductance 1250 nH. Since the magnetic losses of the transformers were negligible compared to the ohmic ones, the pot has been machined in order to increase the specific inductance up to 6000 nH. The primary and secondary coils are wound on individual coil formers. \( L_{F1} \) and \( L_{R1} \) inductances are made of one-layer joint turns while \( L_{F2} \) and \( L_{R2} \) are coiled irregularly on seven-section coil formers, which reduces their parasitic capacitance. The two transformers are placed inside soft iron cylindrical boxes which act as shields against external magnetic disturbances. The transformers’ main mechanical and electrical characteristics are listed in Table II.

The active part of the circuit can be split into three blocks: a differential input low-noise amplifier (\( T_1^1, IC_1 \)), an intermediate stage (\( IC_2 \)), and an output buffer which

\[ \text{T A B L E II Mechanical and electrical characteristics of the input and feedback transformers.} \]

\[ \begin{array}{|c|c|c|}
\hline
\text{Transformation ratio} & \text{Input transformer} & \text{Feedback transformer} \\
\hline
N_F = 30 & N_R = 1/20 \\
\hline
\text{Primary coil} & 10 turns & 160 turns \\
\text{Cu wire, } \Phi = 1 \text{ mm} & \text{Cu wire, } \Phi = 0.25 \text{ mm} \\
R_{F1} = 34 \text{ m} \Omega, L_{F1} = 575 \mu \text{H} & R_{R1} = 3.8 \text{ m} \Omega, L_{R1} = 180 \text{ mH} \\
\hline
\text{Secondary coil} & 30 turns & 8 turns \\
\text{Cu wire, } \Phi = 0.12 \text{ mm} & \text{Cu wire, } \Phi = 1.6 \text{ mm} \\
R_{F2} = 47 \Omega, L_{F2} = 515 \text{ mH} & R_{R2} = 8 \text{ m} \Omega, L_{R2} = 465 \mu \text{H} \\
\hline
\end{array} \]
feeds the resistive low-impedance feedback path. The field-effect transistor (FET) differential common drain pair ($T^+$) uses a parallel association of $2SK170$ Toshiba transistors$^7$ (two FETs on each side), which both reduces the input voltage and current equivalent noise densities and whose overall gain is fixed to about 40 by the feeds a low-voltage noise LT 1028 operational amplifier$^\dagger$ provides a low-input capacitance ($\approx 20$ pF). This stage the unity gain buffer in order to roughly null the dc current in the return path. Finally, the $R_3, R_S, C_2$ lag circuit ensures the overall preamplifier stability at high frequency. The open-loop and input-output gains are $17$ and $10^3$, respectively, in the high-frequency range. Furthermore, the amplifier noise density $e_{nA}$ settles to about $1.4$ nV/$\sqrt{\text{Hz}}$ at frequencies higher than 150 Hz (crossover point), while $i_{nA}$—which increases with frequency—is close to 50 fA/$\sqrt{\text{Hz}}$ at 100 kHz.

The transformer and amplifier characteristics have been adjusted in order to operate between 10 Hz and 100 kHz. Figure 4 shows the preamplifier frequency response as a function of the source resistance $R_S$. For the critical one, the $-1$ dB bandwidth extends from 7 Hz to 100 kHz. The common mode gain was also measured; up to 15 kHz, it increases at a 20 dB per decade rate and settles to $-103$ dB at the line frequency. Beyond 15 kHz, the common mode gain increases at a rate of 40 dB per decade.

The preamplifier experimental noise density is plotted in Fig. 5 for a $R_S = 0.5$ $\Omega$ source resistance. At high frequency, the noise level lies close to 100 pV/$\sqrt{\text{Hz}}$ when $R_S$ is placed at 300 K and it lowers to 80 pV/$\sqrt{\text{Hz}}$ if $R_S$ is cooled down to 77 K. From these measurements, it be-

![FIG. 4. Preamplifier transfer frequency response as a function of the source resistance; $R_S = 0.5$ $\Omega$ is the critical value.](image)

![FIG. 5. Preamplifier input equivalent noise density from 10 Hz to 100 kHz for a $R_S = 0.5$ $\Omega$ source resistance at 300 K [curve (a)] or at 77 K [curve (b)]. The (c) horizontal line marks the 77-K noise level of a 0.5-$\Omega$ resistance.](image)
comes $e_n | \omega \rightarrow \infty \approx 65 \text{ pV}/ \sqrt{\text{Hz}}$, in quite good agreement with the calculations from (14). In the high-frequency limit, the current noise $i_n$ goes as $i_n / \sqrt{f}$ in (15) and increases with frequency; it settles to about 1.5 pA/ $\sqrt{\text{Hz}}$ at 100 kHz. This leads to an overall energy resolution about $\sqrt{2}$ higher than that expected in an ideal case. Elsewhere, as discussed in Sec. III, $e_n$ increases as $\omega^{-1}$ at low frequency ($f < 120 \text{ Hz}$), which shows that the preamplifier open-loop low cutoff frequency must also be taken into account in the design.

8. Linear Technology Corporation, 1630 McCarthy Blvd., Milpitas, CA 95035-7487.